Spring 2010 Math 245-2 Exam 1 Solutions

One quarter of students scored 64-76, one quarter scored 76-82, one quarter scored 82-88, one quarter scored 88-98. In particular, the median was 82, the low was 64 (everyone passed, hurray!), and the high was 98.

Problem 1. For each of the following, indicate which, if any, of the following definitions apply: proposition, compound proposition, universal conditional proposition, quantified proposition, and/or predicate.

a. The sun orbits the earth.
   Proposition only.

b. If the sun orbits the earth then the Saints won the Super Bowl.
   Proposition and compound proposition.

c. $x > 3$
   Predicate only.

d. $\forall x \in \mathbb{Z}, (x > 3) \to (x > 5)$
   Proposition, universal conditional proposition, quantified proposition.

e. $\exists x \in \mathbb{Z}, (x > 3) \to (x > 5)$
   Proposition, quantified proposition.

Problem 2. Starting with the base statement "Every polynomial with at least two zeroes is continuous.", supply each of the following. Simplify your answers.

a. contrapositive
   Every polynomial that is not continuous has at most one zero.

b. converse
   Every continuous polynomial as at least two zeroes.

c. inverse
   Every polynomial with at most one zero is not continuous.

d. negation
   There exists some polynomial with at least two zeroes that is not continuous.

Problem 3. (1.10 in text) Write the negation of the proposition $\forall x \in \mathbb{R}, 0 \geq x > -5$. Be sure to use De Morgan's laws to simplify your result.
Note: $\mathbf{0 \geq x > -5 \equiv (0 \geq x) \wedge (x > -5)}$**, hence the answer is** $\mathbf{\exists x \in \mathbb{R}, (0 < x) \vee (x \leq -5)}$**.**

Problem 4. (4.13 in text) Write the negation of the proposition $\exists x \in \mathbb{R},\ \forall y \in \mathbb{R}, x + y = 0$.
$\mathbf{\forall x \in \mathbb{R},\ \exists y \in \mathbb{R}, x + y \neq 0}$.

Problem 5. (3.3 in text) Use a truth table to determine whether this argument is valid:

$$\begin{array}{r} p \to q \\ q \to p \\ \therefore\ p \vee q \end{array}$$

p	q	$p \to q$	$q \to p$	$p \vee q$
T	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**
F	**T**	**T**	**F**	**T**
F	**F**	**T**	**T**	**F**

The fourth row has the hypotheses true, but the conclusion false; hence the argument is invalid. Side note: the second and third rows can be eliminated because they do not satisfy the hypotheses $\mathbf{(p \to q,\ q \to p)}$**.**

Problem 6. (1.13 in text) Use a truth table to determine whether $(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$.

p	q	r	$p \oplus q$	$(p \oplus q) \oplus r$	$q \oplus r$	$p \oplus (q \oplus r)$
T	T	T	F	T	F	T
T	T	F	F	F	T	F
T	F	T	T	F	T	F
T	F	F	T	T	F	T
F	T	T	T	F	F	F
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

We compare the fifth and seventh columns; they match, hence the answer is 'yes'.

Problem 7. Find a proposition using only combinations of $p, q, \vee, \sim$, that is logically equivalent to $p|q$.
We know $\mathbf{p|q \equiv \sim (p \wedge q)}$, which by De Morgan's law is equivalent to $\mathbf{(\sim p) \vee (\sim q)}$. This can also be proved with a truth table.

Problem 8. Convert the number 256_{16} to base two and to base ten.
For base two, we can go one hex digit at a time, $\mathbf{2_{16} = 0010_2, 5_{16} = 0101_2, 6_{16} = 0110_2}$, so $\mathbf{256_{16} = 001001010110_2}$. For base ten, $\mathbf{256_{16} = 2(16)^2 + 5(16) + 6(1) = 512 + 80 + 6 = 598_{10}}$.

Problem 9. Fill in the missing justifications, including line numbers, for the following proof.

1.	$(p \wedge q) \vee r$	hypothesis
2.	$t \rightarrow \sim s$	hypothesis
3.	$r \rightarrow s$	hypothesis
4.	$\sim s$	**modus ponens on step 2 (note that tautology t is always true)**
5.	$\sim r$	**modus tollens on steps 3,4**
6.	$p \wedge q$	**disjunctive syllogism on steps 1,5**
7.	$\therefore p$	**conjunctive simplification on step 6**

Problem 10. Give a complete proof (from hypotheses to conclusion) that uses both universal modus ponens and conjunctive addition.
Many solutions are possible, of course. A short one is:

1.	$\mathbf{\forall x \in D, P(x) \rightarrow Q(x)}$	**hypothesis**
2.	$\mathbf{a \in D}$	**hypothesis**
3.	$\mathbf{P(a)}$	**hypothesis**
4.	$\mathbf{Q(a)}$	**universal modus ponens on 1,2,3**
5.	$\mathbf{\therefore P(a) \wedge Q(a)}$	**conjunctive addition on 3,4**